

PROBLEMS OF THE WEEK

Action Research

ABSTRACT

Many math students feel intimidated by the phrase "Problem solving". They avoid word problems and feel anxiety when asked to solve one. This story is my journey from word problems as one thing on our "to do" list to word problems as everything on our "to do" list.

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Table of Contents

Chapter 1 – All about me	. 2
Chapter 2 – The idea for my action research begins	.4
Chapter 3 – A trial run: the first bump in the road	. 5
Chapter 4 – The official Action Research proposal	.7
Chapter 5 – What some of the literature tells me about problem solving in mathematics	. 8
Chapter 6 – Getting to know the class	10
Chapter 7 – The start of a focus group	21
Chapter 8 – Previous "feel good" moments from the student's perspectives	23
Chapter 9 – Students lose their enthusiasm for math	25
Chapter 10 – The final focus group: Background information	27
Chapter 11: A check-in with imagination	29
Chapter 12: PoW #1 – Our first "Problem of the Week"	31
Chapter 13: PoW #2 – Any progress?	36
Chapter 14: PoW #3 – Changing direction – cruise ships	11
Chapter 15: My problem - feeling scattered	45
Chapter 16: My solution - Make it worse	17
Chapter 17: PoW #4 – The build up	54
Chapter 18: PoW #4 – Question and results	59
Chapter 19: More in class problems6	52
Chapter 20: PoW Reflections - PoW #1 through #4	56
Chapter 21: I get it	78
Chapter 22: Time slips by	79
Chapter 23: What's next?	35
Chapter 24: Three last items	39
References) 1

Chapter 1 – All about me

I didn't begin as a math teacher, but I am now. And I love it. I have my dream career and I never saw it coming. In my early twenties, after much deliberation and hesitation, I began the education program in Red Deer, Alberta. Even though I loved what I was learning and did really well, I still had this nagging feeling that I should "do better". I could be a dentist, a doctor, or even a physiotherapist. I moved to BC and enrolled in courses that would lead me to a Bachelor of Science, which would then allow me to pursue these other careers. I still did well, but never felt the same interest or passion as I felt in my first year Education courses. After several more moves, each veering towards and away from a teaching career, I found myself overseas, teaching English and Math in both tutoring and classroom environments. I loved being in the classroom and, after many years of denial, finally accepted that this was where my heart belonged. Once I made my decision, the inner turmoil of conflicting self-expectations seemed to stop almost immediately. I re-read one of my first year textbooks "The First Days of School" and the author's dedication seemed to resonate within me:

Dedicated to my father and mother, who wanted me to be a brain surgeon. I exceeded their expectations. I became a scholar and a teacher. (Wong & Wong, 1998)

After I completed my Bachelor of Science, I was accepted into the post-degree professional program at the University of Victoria, graduated with top marks, and began my career eager to become an amazing middle school teacher. Instead, I ended up in high school, teaching a mixture of science and math courses. I began to feel neutral about teaching the science courses and my interest unexpectedly shifted towards the math department. These courses were challenging, intense, and fast paced. It was gratifying to see the progress of students, and the "aha" moments of both the students and myself were much more visible and rewarding to experience. When the opportunity to teach mathematics full-time arose, I jumped at this chance. And here I am, twelve years later, still as excited and interested as I was at the beginning of teaching these courses. Some courses I've now taught 25 times, but I still don't have consistent unit plans that I use every year. Every time I teach a course, I'm tweaking the layout, the activities, and all the other components that are involved. I am still learning and feel like I've just scraped the surface of the infinite curricular connections within each course.

In retrospect, it seems natural that I ended up teaching math. In University, there were only two courses in my entire undergraduate program that I was truly engaged in. One was a mathematics course involving permutations and combinations. It wasn't the course I received top marks in, but I looked forward to it and enjoyed doing the assignments, even when I wasn't able to successfully complete every question. The other course I enjoyed, perhaps even more, was a Biology course titled "Critical Thinking". The major project involved finding a published research paper, analyzing it for errors in process and flaws in reasoning, and then to provide an alternate research proposal that would remedy these faults. It was very difficult to do, but I felt so much pride when I finally found an article – the chosen article described an experiment regarding the behaviour of shore crabs. While the original experiment had a strong underlying idea, there were shortcomings in the design and in the analysis. I then identified what turned out to be many subtle flaws, and used background knowledge and research to propose a different approach which would result in a more efficient and effective plan to study the crab's behaviour. Even though I don't remember the details of my solution or even the original experiment, I will always remember the feeling of being challenged and having to use creativity and logical reasoning to solve a difficult problem.

Chapter 2 – The idea for my action research begins

While I enjoy challenging problems, this is not always true for my math students. I have noticed that many students find word problems particularly daunting, and when given the option, will choose not to solve these questions. Many are resistant to working on one question for a long period of time and seem to lack grit and persistence to follow through when challenged. Last fall, I was inspired by the keynote speaker, Fawn Nguyen, at the annual BC Association of Math Teachers conference. Like me, her journey began in science, but transitioned to include math. In part of her speech, she talked about her "Problems of the Week". She would assign one word problem every week, collect it, mark it as either 10/10 for clear concise work with a viable solution, or 1/10 for incomplete work. The students then would have one week to collaborate outside of class time, and resubmit for full marks. Most of the problems she used were taken from a website called "The Math Forum". She distributed several sample problems for us to try and excited by the prospects of what my students could do, I took them back to my class immediately.

Chapter 3 – A trial run: the first bump in the road

With the first semester already halfway complete, I chose to give one "Problem of the Week" to all my students (Grades 9, 10, 11). I used Fawn's sample questions that used logical reasoning and number sense which were not tied explicitly to a specific curricular content heading- all students should have been able to solve them. When I assigned the question I told the students that it was to be done outside of class time and I wanted them to collaborate with each other, with other friends, and with their families. I really wanted this problem to be part of the dinner hour conversation. I also told them that I had not yet solved it and therefore, they should not "spoil" it for me if they figured it out first by blurting out the answer. The students seemed to perk up at this and enjoyed handing in their papers face down so I couldn't see the answer. They discussed results in quiet voices and on several occasions I heard comments such as "don't say the answer out loud so we don't wreck it for her" or "tell me how you did it but not what the final answer is". Once I solved the question, several students started coming in outside of class time to discuss their strategies. The first problem was "Noah's Ark" and I was ecstatic about the results. I saw at least five different strategies being used, many of the students using 3-4 pages of paper to show their solution to this one question. Some reported working three hours on it. I had never seen students work so hard on a single question. While handing back the student's work, a discussion began about solutions available online. I had naively not checked in advance but now knew that the solution to "Noah's Ark" was quite readily available. They seemed surprised at my reaction. I told them that they had robbed themselves of the experience of working through this tough problem, however, if they had genuinely tried to solve it prior to looking online, it was okay. As long as they understood the online solution and didn't just blindly copy it down, I would rather have them do that than nothing at all. At this point, some students confessed to looking it up on the internet. Some stated that it helped them to understand and others stated that they found it too confusing and went to a peer for help instead.

Unfortunately only about 60% of the students handed the first assignment in, and it went downhill from there. By the end of the term, after five "Problems of the Week" had been assigned, the completion rate was down to about 20% and minimal work/ explanations were being done. I was frustrated and needed to re-

vamp my approach. Why was the completion rate so low? Which students were actually doing the problems - were they the "top students" who had a good work ethic anyway? It was mandatory at the start but had become optional and I used it as the "tipping point" to bump students who were within 1-2% away from the next letter grade. Did this cause the students to only complete them if they were mark driven? I wanted to emphasize the joy of solving a complex problem, but had I wrecked it by making it optional but also worth marks? And then there was the problem of approximately 140 students that I was responsible for. Two of my classes were linear, in which every second day we had class, and the other three were semester, so I saw them every day. The due dates became inconsistent and difficult to monitor which resulted in vague expectations of deadlines. I knew better but felt like I was drowning in the paperwork of marking this assignment in amongst the rest of everything else I needed to assess and give feedback on. Additionally, I found myself reflecting on my choice of problems. I had given all grades the same question partly out of convenience for marking and partly because I wanted to encourage collaboration between grades and classes, but I wondered if this deterred some students from completing the work because it didn't directly correlate with what we were learning in class. Were these problems increasing my student's mathematical confidence and curiousity or did they do the opposite? One of my students approached me at the end of the semester and asked if I was going to assign the second semester a different set of problems. I told him "yes, I think I will" and he then asked if it was okay if he came in once a week to get a copy so he could figure it out too. Of course it would be okay! I found his interest and enthusiasm inspiring – now I just needed to get more students to feel this same way. Like the shore crab research paper, I felt like there was a really good idea underneath all the mess, I just needed to sort it out and find a better solution that would minimize the flaws in my application of this "Problem of the Week".

Chapter 4 – The official Action Research proposal

I elected to focus my action research on my second semester's new group of Math 10 pre-Calculus students. The course itself is perhaps my favorite curriculum as I feel both comfortable and challenged by the content. I have the most available resources, am the most organized with regards to course planning, and am very aware of the subtle connections between skills that are used throughout the course. The following statement is from my school district application to perform the action research:

Under the discourse of Imaginative Education, I want to explore how to improve the mathematical confidence and depth of understanding in my students using cognitive tools. Focusing on problem solving, I will examine connections between curiousity, determination, enthusiasm, and perseverance compared to overall confidence in mathematics.

With approximately 4 weeks to gather data and make observations, I immediately signed up for the annual membership to access "Problems of the Week" as recommended by Fawn Nguyen (The Math Forum, 2017). Even though my action research was focused on my grade 10's I still wanted to provide this experience of solving assigned problems of the week, or PoW's, for my other three courses as well. I decided to choose questions that were grade specific and used either skills we were either currently using or skills that had been learned in past grades that would be applied in future lessons. Each course would get a different PoW. This selection process was far more challenging and time consuming than I had anticipated. I wanted questions that would complement the in-class lessons, were challenging to complete, but were still achievable.

Chapter 5 – What some of the literature tells me about problem solving in mathematics

Find the story.

Using the narrative encourages affective engagement with the content being taught (Egan K., 1997). I realize that making historical connections will add depth to my lessons but I am still developing my own appreciation of this aspect of the content. Since students will gain a better appreciation of mathematics if they are exposed to the historical story of the tools that we are currently using (Meavilla & Flores, 2007), I will try to add depth and historical context that will help students to feel emotionally connected to the assigned problem of the week.

Additionally, the sequence of the chosen problems is integral to the bigger story. My goal is to start with the abstract and, through every experience, work towards systemic understanding. The story is not in the details of each assignment but instead of the entire course as a single unit.

> To wonder is to acknowledge one's ignorance, not in a state of despair or passivity, but in the pleasurable pursuit of further knowledge

> > (Sinclair & Watson, 2001)

Find the wonder.

Under the discourse of Imaginative Education, I feel that the sense of wonder, along with emotional engagement, is at the base of this educational approach. Wonder in mathematics can be expressed as:

- a) the desire to understand and/or prove an observed relationship
- b) the desire to test extensions, variations, and possible generalizations (Zazkis & Zazkis, 2014)

Learners... need to be inducted into the wonder of mathematics, to experience wonder vicariously through the teacher (including the stages of pleasure and frustration that sense-making requires) and, more urgently, to set aside the illusion of mathematics as systematic knowledge so complete that there is nothing more to expect.

(Sinclair & Watson, 2001)

I want my students to truly understand the concepts that what we cover in class, I do not want them blindly following a series of steps. The steps are important, however, there is more often than not, more than one series of steps that could be used. I also want to inspire my students to ask more questions and to feel curious about this subject. Silver and Stein (1996) state that when students are taught in a way that focuses on the conceptual understanding and problem solving, in addition to skill development, they will demonstrate a deeper understanding and application of the topics.

Boaler (2002) discusses how teachers need to provide differentiated work for their students. This can be done either through differentiating by task, in other words providing different work, or it can be differentiated by outcome, in which students are given the same activities but can approach them in a variety of ways. In the past I have tried to differentiate by providing different work and found it difficult to manage and not effective given the time constraints of things that need to be done every day. Instead, I will aim to differentiate the problems of the week by selecting questions that can be solved in a variety of ways. This should encourage more students to work on these assigned questions. Additionally, since students will develop a meaningful understanding of mathematical ideas when they are engaged in challenging mathematical tasks (Silver & Stein, 1996), I also need to choose questions that are not too easily solved. The following expression came from an article on music instruction, however, I find it rather catchy, and since music is very mathematics based, I have chosen to include it.

> The practice of skills can become more interesting as well as challenging when incorporated into creative exercises initiated by student or teacher; further, practice enhances the internalization of skills that allow for greater versatility in improvising and composing. (Kuzmich, 2013)

Chapter 6 – Getting to know the class

At the start of every course I have students complete a Graffiti activity. They travel in groups and spend several timed minutes moving from one station to the next brainstorming lists. If they strongly agreed with something that someone else had written, they would put a star by it. We didn't have much time this particular class, so each group only spent approximately 2 minutes at each station. The student's enjoy moving around the room and getting to know each other better and I enjoy seeing their responses. It allows me to get a sense of the class dynamic right from the start. In this group there are 30 students, however, one student was away on holidays for almost the entire duration of my action research.

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Careers that use mathematical skills (reasoning)

- practice questions S- practice test ask for help (teacher, Family) A ask the - redo topics er questions - go over notes - Know the topic - look for helpful videos How to. - do your hwat prepare for assessments - Study Everyday for any (tests, gurrages, final examp) hour Study with a friend the - make flashcards the

How to prepare for assessments



Test taking strategies (during the test)

What does doing well in this close schuelly mean ? · Being able to opply when in class tork real life 1 · Setting A's that · getting a good grade A · improving on things you weren't good at before. · Learning New # 2 Things Learning how to solve problems and not just memorizing it. · Be proud of what you got - actually understand what volve doing # \$ · Being able to apply the stuff learnt in class to real world. ·Not having to ask questions

What does "doing well" actually mean?

Completing all the assignments At Paying attention & Paying attention & Showing up to class Helping others ## Son time Being respect full # getting good grades # # I had funder # -Understanding Concepts * A -taking away from the Class,* - being able to use it outside of class - WOrk -being able to remember topics

2nd station re: What does "doing well" actually mean?

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& Practice	How to Do Hall fish & * IN THE CLASS BE PROPORED
come at lunch for extra help	HE ISTEN Concepts ready to learn

How to "do well" in this class?

Best Back & White Answers(either wrong or right) Learning Life skills Alaebro enorizing XXXXX Vactions muHiplication divition & Challonges your brain RACK Stboww, Percentage everything AAAA Final exams (tests communicating w/ friends & The var it changes how you fee life Math

Best part of math class vs. not so favourite part of math class

Chapter 7 – Getting to know the students

At the start of every course, after we have done the graffiti activity, I put the following quote on the projector and give their first assignment.

The process of writing requires gathering, organizing, and clarifying thoughts. It demands finding out what you know and don't know. It calls for thinking clearly. Similarly doing mathematics depends on gathering, organizing, and clarifying thoughts, finding out what you know and don't know, and thinking clearly. Although the final representation of a mathematical pursuit looks very different from the final product of a writing effort, the mental journey is, at its base, the same – making sense of an idea and presenting it effectively. (Burns, 1995)

The first assignment I give is an essay. I leave it very open ended with only several criteria. It must be handwritten and the title is "My Math Career". They have one night to write it and I prefer that they write from their heart than worry about formatting, grammar, or proper thesis statements. The length is up to the student and even the direction they take is up to them. Some students choose to write a narrative of the topics they have learned since kindergarten, some talk about the people who have influenced them – both in positive and negative ways, and some students discuss their future ambitions. Some will submit a one page double spaced minimalist essay and others will hand in 8 pages of single spaced writing full of rich dialogue and ideas. They are always interesting to read, but in the past I have found that it's hard to make connections between what the students have written and who they are since I don't even know their names yet. However,

I have noticed that many students describe a love/hate relationship with math. Common statements include phrases such as:

- $\ensuremath{\mathfrak{S}}$ I used to love math when I was younger, but then it changed
- ☺ I struggle in this subject
- $\ensuremath{\mathfrak{S}}$ I've never been good at math
- (a) Math used to be easy and fun but then it got more difficult and I don't enjoy it anymore
- ☺ I don't like math but know that it is important
- ⊗ I am not interested in this subject
- ③ I am not good at math and I blame my family/teacher/tutor/etc.
- ③ I don't like math and doubt that I will use it after I graduate

There are usually positive papers as well that include statements such as:

- ③ I've always loved math
- © I'm strong in this subject
- © I enjoy solving difficult questions
- © I think it is important
- © I didn't use to be good at it but then in grade (fill in the blank), I started to understand it
- I plan to use it in a future career

From essays in the past, my perspective is that that these positive papers seem to be the minority. I expected the same sort of results this semester and planned to use these essays to narrow down a pool of students to focus my action research on. I would choose students who explicitly expressed negative attitudes, low ambition, or disinterest in the subject. However, I didn't get what I expected.

"MY MATH CAREER"

Some highlights from the student essays:

"Throughout elementary I loved the subject.... As soon as high school hit, that's when things became real...now it is a lot more difficult."

"I changed schools in elementary because I did not like the teachers... the influence of a teacher changed my entire thought on math: making me improve from a "C" to an "A" student."

"In elementary I think I was below average because I struggled with math. Learning new concepts was difficult because I was a slow learner... even though I have improved, I still struggle"

"(In) high school, everything became more difficult... I started getting B's and C's instead of A's which was sad for me and made me lose the interest and joy of math... I am a very ambitious student who doesn't like not being good at average level classes... so I got extra help from my dad and a tutor and it worked"

"I have a knack for numbers...I always try my best... that has led to my success"

"Not trying will not make me feel good knowing I could have done much better"

"Although I feel that a vast majority of what's taught in math class is irrelevant to the rest of my life... I do enjoy trying to understand it and work out problems and it's fascinating to break it down and understand its code"

"I like getting challenging questions because it challenges me to think harder...learning what you're doing and understanding is more important than getting an "A"... My role model has always been my dad"

"Math... is something I've seen as intimidating for most of my life. I've never really been good at it. However, I have a new found respect and admiration for math"

"...I feel that I have a choice as to where I want to take my math skills. I could just put a minimal amount of effort into it and barely pass. On the other hand, I could move forward and excel at it... In elementary school I really didn't care about being good at this subject"

"Math is one of the subjects I've always been good at"

"My mom has been my main supporter... I know I can do better...I strongly believe that math is the foundation to having a successful life"

"I love math because it is black and white with answers...I like the rules and the structure but hated showing my work... challenging my brain makes me happy because it shows me what I am capable of"

"I'm looking forward to pre-Calculus 10"

Not one of the essays seemed to jump out at me. Students who claimed to struggle in this subject, took ownership for not coming in for extra help when they needed it. Few seemed to talk about grades and instead focused on understanding or not understanding topics. In fact most of the essays seemed to reflect students that were excited to learn and eager to improve. What had happened? Did I stumble upon a group of keen students that were "the dream class"? Did they only write what they thought I wanted to hear? I didn't explicitly tell them that I was reading these essays with my action research in mind, but had this influenced their writing? I decided that this was not the case since I had presented my research to them as primarily a project in which I was reflecting on my own lessons. I decided to wait a week, get to know the students a little better, and then re-read the essays with a closer eye. It was in the re-read, which I have never done before, that the subtle comments stood out. I now had a potential focus group of students to focus my action research on.

"MY MATH CAREER"

Some highlights from the essays of the potential focus group:

"My experience in math has been a hit and miss situation...I do enjoy most parts of math but I despise Pythagorean's because I just can't grasp the concept. I also don't care for challenging variable questions but I hope that this changes because I really do enjoy math...Math is a great and useful subject... I am a hard working student and hope to improve my math skill."

"I'm not that good at math but, if given further time and examples, I will get it... My other family members are really good at it, and I as I notice myself being slightly uncomfortable with it, it made me feel like I was not good enough... I am always willing to learn and or discover new things. Math may be sometimes harsh, but hopefully I can get through it."

"I have never been very good at math due to inability to get concepts fast... I hated subtracting... (Then I was) introduced to fractions and my hate was placed elsewhere.... I was introduced to harder and harder math until math became my least favorite subject. It's never been my favorite and never will be... when I get out of high school I want to be a pilot. Though I've never been the best at math I do my best because I know it will help me in the years to come"

"The moment I started (to) learn math, I kind of didn't like it at the time because I didn't get it at first... teachers just had to make it more difficult every single year... but I got through it. But math isn't always the easiest thing – it's a challenge for a reason, you will find a question at some point and you won't be able to solve it because you may not know what formula to use... In my path it was a hard journey but I made it back onto the path"

"Ever since grade 1, I was always scared of numbers because I always hated math"

"Math is my least favourite subject. It is hard for me to organize my thoughts. I also have trouble figuring out questions and trying to solve them. I am simply not good at numbers... I concentrate a lot and try to understand the topic we are doing. Despite this, math is unfortunately not a skill of mine. I would like to be able to fully learn the comprehension of Math, especially as it is my worst subject."

Chapter 7 – A written survey

At the start of the second week of the semester, I gave a survey to all students that included both closed-

ended questions as well as open-ended questions. One of the open-ended questions was as follows:

What is your least favorite part of math? Tell me a little about this.

I have sorted all student responses into categories and common themes:

Marks/assessments:	>	When a surprise test appears. It makes me anxious thinking about the next		
		one I constantly stress about it and the worst part is when you try for at		
		least 3 out of 4 and you only get 2.5 and you know this stuff from before.		
Homework	6	The homework that is given when I have homework for all my other		
Homework.		classes		
	~			
Specific topic: Dislike		Integers and fractions because I get confused		
or lack of		Pythagorean's theory because I always get it mixed up		
understanding		Fractions because it is sort of hard to do quickly		
	>	Algebra because it is hard for me		
Specific topic: Feeling	\triangleright	Learning topics that I feel will not help me in the future (example:		
of irrelevance		Pythagoras). I have a problem with the education system and learning		
		things that don't pertain to me is an issue. Although I do find it interesting		
		to understanding those learning outcome, I wish it wasn't as in depth, if		
		that makes sense		
		I love math but I guess graphing. I never took to graphing. I would rather		
		just solve and answer not graph		
		Surface area. I think it is one of the most useless skills in math as most of		
		us will not use it later in life.		
Feeling unsure:	\triangleright	Doing questions that I don't know the steps to		
	\triangleright	Not understanding tasks or being behind		
	\succ	Doing things that I have no idea how to do		
	\succ	When I have troubles picking up what we're doing		
Showing work or	\succ	I don't really like when it takes me a long time to do one question		
reasoning/time to	\succ	Doing all the work that has to be shown		
solve a question:	\succ	Surface area and geometry because I have never been good at it and		
		dislike showing all of the steps		
	\succ	I don't like calculating surface area. It takes many steps and can take over		
		5 minutes		
Word problems:	\succ	Word problems because you have to verify certain components to find		
-		your equation and then solve		
	\succ	Some word problems because sometimes I don't fully read it correctly and		
		as slow as I should		
	\succ	Word problems because I have a hard time following up with the question		
	\succ	Solving word problems		
	\succ	Word problems because they get confusing and I do really bad at them		
	\succ	Word problems and multi-step questions are my least favourite because		
		(they) require you to use multiple skills		
Other:	\succ	I cannot think of anything I dislike		
	\triangleright	My least favorite part is the perspective that most people have on it. that it		
		is not a good one		

Chapter 7 – The start of a focus group

While I plan to document data from all students, I would like to focus on a subset of students within the class. After comparing the survey results to the essays, I narrowed down my list of focus group candidates to four possible students. From this point forward I will refer to them as Student A, B, C, and D. Included on the survey was the following closed-ended question:

Which type of math questions do you prefer (check off any that apply)

I intentionally had overlap between "descriptions" using words/ vocabulary that I have heard students use. For example: challenging questions are often questions that require the use of multiple skills, have multiple steps, and also have a range of possible strategies that could be used. I have tallied up some of the results and have focused on descriptors of questions that will be used on the upcoming Problems of the Week, or PoW's.

	YES Class Results, not including Focus group: 23 students wrote survey		YES Focus Group Results: 4 students	
Type of Question:	Selected /23	Percent that prefer these questions	Selected /4	Percent that prefer these questions
Questions that I can work by myself on	16	70%	2	50%
Multi-step questions	15	65%	0	0%
Questions that can be solved in more than one way	14	61%	3	75%
Work with a partner or in small groups	12	52%	3	75%
Questions with step-by-step solutions	12	52%	1	25%
Challenging questions	11	48%	2	50%
Questions that require the application of multiple skills	11	48%	0	0%
Questions with more than one answer	9	39%	2	50%
Questions with no solution	7	30%	1	25%
Questions that are unfamiliar and I am not immediately sure how to solve	4	17%	0	0%
Questions that are familiar and I know how to solve	3	13%	4	100%
Word Problems	3	13%	0	0%

I was surprised that working independently had a higher selection than working in partners or small groups, however, I suspect that the wording on the survey impacted the student's selection. Instead of "work with a partner or in a small group" the survey listed "partner or team challenges" and my intended category and the student's interpretation could easily be interpreted as different things.

Regarding the focus group, no students selected multi-step questions and I expected these results. I was surprised about the results regarding familiar questions. The focus group unanimously agreed that they preferred familiar questions that already they knew how to solve. Compared to the rest of the class this 100% seemed extreme versus 13%. This seems to imply that the focus group students prefer to work within a comfort zone of familiarity more so than the other students in the class.

As expected, "Word Problems" did not rank particularly well, but I predicted that it would have been higher than "questions with no solution". None of the focus group members selected "Word Problems" and Student A commented "sad – no thank you" when asked how she felt about this topic. Being at the bottom of the list affirms that this is something I want to work on with my students. I need to help them find the joy and intrigue in solving the mystery of word problems.

Chapter 8 – Previous "feel good" moments from the student's perspectives

The last question on the initial survey connected to emotional engagement:

Describe a "feel good" moment you've ever had in any math class

Again, I categorized their responses into common themes:

Marks/Assessment	 Getting 100% on a test I studied really hard for in grade 9 When I get A's on tests When I had a test I walked in very prepared and the test was so easy because I knew everything When I do really good on tests
	 When I get an A on a test When I got an A in 4th grade and my teacher didn't really like me
Confidence/ Understanding	 When I understand a tough problem When I understand what I'm doing exactly Solving a question that you got wrong at first When you solve a task correctly Last year in grade 9 when math class was fun because I well understood everything When I was able to fully comprehend the lesson when we first did notes on it When I finally understand a challenging question When I'm working on BEDMAS questions When I finally solved a question after 3 minutes and getting it right
Success/Feeling of pride, while others are struggling	 Knowing the answer when no one else in the class does Whenever there's a difficult problem that I can solve both other people are struggling with it makes me feel good about my math skill When I solve a question that nobody in class got When there was a hard question that needed to be solved and not many people solved it but I did When I helped my partner understand something she doesn't get
Responses from the focus group	 When I got an A in a computer course Doing multiplication When I know the answer and how to solve it and get the answer correct When I do something correctly

These results give me an insight into the students motivating factors. For some students they seem to be mark driven and others seem to be more focused on their own feelings of success and understanding. I found it intriguing that many students described their "feel good moment" by comparing their success to the 'failure' of their peers. In other words, students felt good when they understood if others struggled. I wondered how I could tap into this and turn it into a mutually beneficial experience through partner work. I already occasionally used giant whiteboards that the students would work together on to solve a question but decided to incorporate this strategy more often. I would give students a question and the boards are large enough that they can either work together or work independently and compare results/strategies. This compliments Vygotskyian beliefs that the "development of thinking is not from the individual to the social but from the social to the individual" (Vygotsky, 1934/2012, p. 38).

Chapter 9 – Students lose their enthusiasm for math

As noticed in the essays, many students made comments along the lines of "I used to enjoy math in elementary but I don't anymore". I wanted to learn a little more about these thoughts. Under BC's curriculum, part of the rationale behind learning mathematics includes the following statement: "Students will experience both success and failure, ultimately learning perseverance while developing confidence and competency in mathematics" (BC Ministry of Education, 2015-a). I feel that part of the pleasure in doing mathematics is in the challenge and in not being able to solve things easily. I enjoy making mistakes and preach to my students that they are a great opportunity to learn and to improve. I feel that if everything was easy it would be boring but I wondered if my students felt the same. I wondered if students lost interest in math because it seemed more difficult, or did it happen when marks started to become the focus of learning, or was it altogether something else.

If you used to love math when you were younger, but don't love it anymore,

Theme: My comments/reflections Complexity/ · It used to be easy but as we get older it gets harder and Perceived harder so I don't like it as much anymore Difficulty This changed in grade 8 because everything became way harder and more complicated questions to solve I didn't like it in elementary but it was easier Topic The unit of 3D objects can easily be very fun • It changed when I learnt about angles and surface area - 3D and exciting to learn in depth - it is my objects personal favorite part of Math8 I never liked math and prefer English and Social Studies Teacher This reinforces my own teaching pedagogy as · It changed by the way that certain teachers taught I encourage and welcome different strategies. • I used to love math before because I was really good at it At times I have felt critiqued by colleagues but now I don't like it because some teachers make you do for not teaching step-by-step processes that it a certain way even if you can do it another way, making it are done the exact same for every question. more confusing I used to love it but now it's okay. This changed because of the teachers I had in grades 5-8. Anxiety -I feel that this one question could be the I didn't love math (in elementary) but I wasn't stressed Grade focus focus of an entire action research – does about it. Now I get stressed about it sometimes. I guess I "math anxiety" begin once we shift the focus started getting overwhelmed by in in grade 6 or 7. This is to grades versus understanding? probably because I actually started caring about my grades. It changed because in the past I used to get A's and once I started grade 8, I consistently get B's Other factors I am curious about math being used as a I loved math because I used it as coping mechanism but "coping mechanism". then I no longer needed it Like to I have always and will always love math Dislike: I always liked math · I used to not like math, but now I don't mind it · I've always enjoy it as much as I do now

when did this change for you? Do you know why it changed?

Not applicable • I used to hate math because of bad teachers but in grade 7 my teacher got me interested and I've like it ever since Focus Group This ties back to the theme of these It changed when it got a little bit more complicated Response particular students feeling more comfortable When it got more complicated with familiar questions. I want to push I used to like it when I was young, but then when high them outside of this comfort zone and school started both my grade 8 and grade 9 math teachers encourage risk taking. I hope that the were really bad at teaching the class upcoming PoW's will begin to accomplish It changed in grade 6 because we started learning harder this. stuff

Chapter 10 – The final focus group: Background information

Our school has over 1500 students in grades 8 through 12. Classes are approximately 80 minutes long and students attend 4 out of 5 periods every day. The class I am observing is in period 2, just before lunch. As the school is on a split bell schedule in which students either attend periods 1-4 or 2-5, this Math 10 block is the first class for most of my students.

By the end of the second week, based on the essay, the survey, and in class observations, I finalized the official focus group to the anticipated four out of thirty students: Student A, B, C, and D. By chance, two of the students are female and two are male. Two of the students are new to this high school as of last September and all four students are in grade 10 Pre-Calculus and Foundations for the first time.

Students A has described herself as not good at math but is willing to improve. She stated that she is not comfortable with unfamiliar scenarios. When working with her table partner, who was randomly assigned on the first day, she has been extremely quiet. She appears to be paying attention and tries to solve questions on the whiteboards but when asked to explain her work or reasoning, the dialogue is short and she appears unsure of her answer. The first week of class was spent on review of order of operations and algebra to isolate a variable. I notice that she is making errors and is struggling to keep up. The table partner she was randomly partnered up with is much more vocal and confident in their work, even if it had errors as well, and I plan to move her in the seating plan, which hopefully will encourage an increase in her participation during partner discussions.

Student B stated that math was her least favorite subject because she was "simply not good at numbers". She stated that she tries to concentrate but that math is still not a "skill of mine". She also stated that is was due to negative teaching experiences in her grade 8 and grade 9 classes that impacted her ability to understand the content. This occurred at another school and she seemed reluctant to talk about it. She is not a fan of word problems and has difficulties understanding what to do. So far, she is very quiet during class discussions and has not been an active participant.

Student C has been vocal in class discussions so far. He seems willing to take risks and make errors in front of his peers, however, in his essay he described himself as not good at math because he "doesn't understand quickly". On several occasions he expressed interest in purchasing the optional supplementary workbook that students have available for extra practice but after one month in the course, he still had not followed up on this. Initially he registered for Apprenticeship and Workplace Math 10, and easier course, however, with future career ambitions of becoming a pilot he will need to successfully complete this course.

"I know that what I just did is probably wrong so I think I need the extra practice" - Student C after completing an assessment on multiplying polynomials. He scored 40%.

He has voiced his interest in improving but has also displayed several instances of negative self-talk when he was unable to immediately solve a question – for example: "I'm sure this is wrong", "This is probably not right since I have no idea what I am doing", and "I'm probably wrong again". He claimed that he began to dislike math in late elementary when it "started getting harder" and I am curious to see if he initially completes the PoW, and if he does and scores 1/10, I wonder if he will show perseverance to resubmit the PoW. By coincidence, his assigned table partner has been absent and I also plan to move him so that he has a consistent partner to work with.

Student D is a male student who initially appeared to be very confident and outgoing, however, the work I have seen to date has many significant conceptual errors and he has been unable to explain any of his reasoning. Instead he shrugs, and with a smile says things like "I'm not really sure why I did that". He seems reluctant to ask for help from me and I am not sure how much he contributes to partner work in class as his random partner is a very strong student. Due to illness, his attendance has been low and by the time we reached spring break, he had missed 6 out of 25 classes – which is almost 1 out of every 4 classes. I am concerned that this will negatively impact his progress as the first month is focused on conceptually understanding the big ideas. He appear to be a struggling learner and this course is fast paced and the content is intense.

Chapter 11: A check-in with imagination

I feel that imagination is at the core of our thinking. It is the reason why we make connections and are flexible in our thinking. It is why we have the ability to think of possibilities.

So what is the imagination? 'It is the ability to think of the possible, not just the actual; it is the source of invention, novelty, and flexibility in human thinking; it is not distinct from rationality but is rather a capacity that greatly enriches rational thinking; it is tied to our ability to form images in the mind, and image-forming commonly involves emotions"

(IERG, n.d.)

Kinds of understanding or Imaginative Education, is a theoretical approach which recognizes that imagination is at the very core of cultural and educational development (Egan K. , 2010). Kinds of understanding represent a series of connected zones that learners pass though as they grow in their ability to think and understand. Each experience in their lives provides an opportunity for growth in each zone and cognitive tools are aids that can be used to strengthen the impact of each experience. As student's progress from one zone to the next, if the zone has been fully developed, they will bring with them some of the power of the cognitive tools from past zones and that zone's resulting depth of understanding. Students can be in one zone while the next zone, and the accompanying group of cognitive tools, begin to take shape. Since the cognitive tools are aids that allow us to tap into, engage, and support the learner's imagination, my goal as a teacher is to use as many cognitive tools as possible in order to maximize and develop the student's current zone of understanding. Many of my students are in the zone of Romantic Understanding (Egan K. , 1997) and the supporting cognitive tools include, but are not limited to: associations with the heroic, collections, humanization of meaning, the literate eye, and sense of wonder (IERG, n.d.). Many, if not all, of my students have passed through Mythic Understanding, but are still connected in different degrees to the cognitive tools of story-telling, binary opposites, and the sense of puzzle and mystery. Some of the students are also using tools of Philosophic Understanding and are attracted to anomalies and meta-narratives (IERG, n.d.). With the PoW's I plan to use a combination of these cognitive tools, both in the selection of the problems, but also in the presentation of the questions. Overall, I want to tap into their sense of wonder to deepen their understanding, interest, and curiousity about mathematics.

To make matters worse, "kinds of understanding" are not neatedged, discrete entities; they overlap, mingle in various ways, cannot be tied precisely to techniques and technologies, are not stages we pass through methodically, may all be used by an individual in a typical day, and so, messily on.

(Egan K. , 2010, Part 3)

Chapter 12: PoW #1 – Our first "Problem of the Week"

So far with regards to curriculum, we had primarily only covered review of order of operations and algebra. We had also begun "Pattern talks", an in class discussion in which we worked together to identify as many patterns and/or formulas to make predictions in visual patterns (Nguyen, 2013). Many of the pattern talks we had done were linear functions. We had also begun to make connections from the pattern talks to graphing. Finally, we had also begun polynomials. Based on what we had covered, the first PoW that I chose

was titled "Location, location, location". I changed the title from the one provided by the Math Forum in order to avoid students doing an online search for a full solution. I also changed details to make the question more concrete that could be solved using a variety of approaches. Students could have solved this question using trial and error, algebraic

Problem of the Week #1 THE "BIG IDEA" BEHIND IT

Operations between polynomial expressions are connected and allow us to make meaning through abstract thinking.

(BC Ministry of Education, 2015-b)

equations and substitution, or even linear graphing. Results could be organized by tables, by cases, or even by graphing. The majority of students used some version of a graphic organizer calculating each case to document their work.

In order to make the collection and marking manageable, I decided that the due date for all PoW's would be on Thursday's. During each Thursday class, the students will upload pictures of their completed work to FreshGrade. This is my first experience with FreshGrade and we are using it only as an online portfolio. In the comment section, students are reflecting on how these PoW's demonstrate their curricular competencies (BC Ministry of Education, 2015-b).

I have decided to assess as follows:

- \square 10/10 = complete/accurate work (either on 1st or 2nd submission)
- \Box 5/10 = two submissions made, but errors are still present
- \Box 1/10 = one submission, significant errors present, written feedback provided to assist in corrections
- \Box 0/10 = not submitted



A local baking company has four stores that sell their baked goods. All four stores lie along the same road as shown:



The distance from store A to store B is 8 miles, the distance from store B to store C is 6 miles, and the distance from store C to store D is 11 miles. There are no back roads that are shorter than staying on the main road.

Based on current sales, store A requires 8 deliveries per week while stores B, C, and D require 6, 7, and 9 deliveries respectively. Each delivery takes a full truck, so the driver can't deliver to more than one store on a single trip.

The company plans to build a new bakery at the same location as one of these stores. Where should they build it so that the total weekly distance traveled delivering the baked goods to the stores is as small as possible?

I designated one wall of our class to post the PoW's for all classes and introduced this bakery problem by watching a clip from an episode of Hidden Houses that discussed the history behind the phrase "the upper crust". I did this to humanize the situation. I thought it was interesting but the students didn't seem to react (Hidden Houses, 2011).



Class Results:

PoW#1 - Week 1: The Initial Submission					
Number of submissions:	Number of student that scored 10/10:	Number of students that scored 1/10 but were very close and on the right track:	Number of students that scored 1/10 and had significant errors:	Number of students that did not submit:	
24 out of 29	10	7	7	5	
This means that 83% of the class made an initial submission. This is higher than it was in the trial run last fall.	This means that 17 the class were either c track	7 students, or 59% of orrect or on the right 	This means that 12 students, or 41% of the class did not demonstrate conceptual understanding of this problem.		
PoW#1 - Week 2: The 2 nd submission					
Number of possible submissions: 19	Number of studer for their 2 8 ou	nts who resubmitted 2 nd time: 1 t of 14	Number of student their 1st 5 ou	ts who submitted for time: at of 5	
	This means that 13 out of 19 students took advantage of this second opportunity to submit their work.				

PoW	#1 –	Focus	Group	Results
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All 4 students made the initial submission, however, they all had significant errors in their process. Only 1 out the 4 students took advantage of the resubmission.

FreshGrade Student Reflections: Focus Group

To be honest. I'm not really sure if my answer is correct. I made a chart of numbers for the number of trips and the stores (it's really messy though, sorry about that) made in the question. I used the chart to help me get my final answer, which was store D. I got store D as the answer, because by the time the deliveries are made, store D would still have goods to deliver. while the other stores have already finished their trips.

I had my older sister help me with this question. She didn't really help me, she kind of just told me to make a chart for the problem. Both of us aren't sure if the answer is correct, though.

I'm pretty sure my answer is incorrect, because I've asked a few people what they got and everyone got different answers. I don't really know which is right, but this problem was a bit hard for me to solve, therefore I think store D isn't really the answer.

Note: This student was not correct and did not submit

Student B

The question was sort of confusing but I was able to get it after thinking about it.

The skills i learned in the class so far has been useful in this worksheet.

I worked alone but after i checked with my friends.

Note: This student had significant errors and did not resubmit

Student C

In the problem given we are given the distances and deliveries per week how I solved the problem by using the most logical answer which is A because it gets the most deliveries per week and would be easier for the driver to start with.

I worked with a friend and compared answers and he as well got A and he showed me how he got his answer

Note: Both Student D and "friend" had significant errors, however, only the friend resubmitted. This student chose not to resubmit.

Student D

"My answer for the question is that the company should build their new bakerv same place as stores A or D, because when they do. they will only travel in one direction and it will be easier for them since they won't have to go circling around. I worked by myself in this problem of the week but if i get it wrong, i would re do it. I feel like I get it but I did something wrong"

Note: This student resubmitted but it still had procedural errors.

Student A

PoW # 1 – Final Data	
Mean Score /10	7.3
Standard Deviation	3.8
Percent of students who submitted on the first week.	83%
Percent of students who took advantage of 2 nd opportunity to resubmit	68%
Percent of students with at least 1 submission	100%

Summary or Problem of the Week #1

I plan to make some revisions in the PoW process for PoW # 2 to try and increase the mean score while decreasing the standard deviation, while choosing a problem that better complements our class work and the level of the course. I am very pleased that all students made at least one attempt and now I need to focus increasing the initial rate of submission on week one.

With regards to the chosen problem, I think that this one was actually too easy and that I had modified unnecessarily. In class we are continuing to work on using polynomials to represent unknown changing variables and we are also working on representing data on graphs. Because of this, I will select the next PoW to reflect these ideas.
Chapter 13: PoW #2 – Any progress?

I turned off the lights, shut the door, and while the students had their heads down with their eyes closed, I

read them the following story:

At the East End of Grand Cayman, far from the island's cruise ship terminal and financial centers, a simple memorial commemorates a maritime disaster that gave rise to a legend about the island's tax free status.

On the evening of February 7, 1794, a convoy of ten British ships en route from Jamaica met disaster on the treacherous reefs of Grand Cayman. The Royal Navy frigate *HMS Convert* led a group of nine merchant ships, six of which were bound for England, three for the United States. The *Convert's* captain, John Lawford, went to bed on that evening believing he had already safely passed by Grand Cayman. Unbeknownst to Captain Lawford, during the night six ships in his convoy sailed ahead of the *Convert* on a course which sent them directly towards the Cayman reefs. At 3:00 AM, one of the ships fired a distress signal and Lawford returned to the deck to discover that his own ship was now bearing down on the reef as well. As Lawford attempted to change course another ship in the convoy collided with the *Convert* driving both vessels onto the reef where they quickly foundered.

All ten ships in the convoy ultimately wrecked on the reef. As the passengers and crew struggled to survive amid the breaking waves, island residents from the East End and Boddentown, having heard the ships' distress signals, paddled out to the reefs in canoes to attempt a rescue. In the darkness and pounding surf, the Caymanians saved 450 of the stranded souls. Amazingly, only six people lost their lives in the disaster.

The heroism of the Caymanians in rescuing the English sailors and passengers fueled a legend that lingers to this day. The story goes that one of the passengers rescued from the wrecked ships was a son of King George III. When the King learned of the island residents' bravery, he decided to reward them by decreeing that the Cayman Islands would forever be free of taxation and war conscription. However, there is no record that any member of the royal family was on one of the ships or that the King ever issued such a decree. Yet the legend is commonly repeated to explain the island's unique financial status.

In 1994, on the 200th anniversary of the disaster, Queen Elizabeth II visited the island's East End and dedicated a memorial to the six victims. On a cliff with a view looking out to the reef where the ships wrecked, a stone monument and plaque commemorate the event. Perched along the cliff adjacent to the monument are six small concrete blocks representing the unfortunate few souls the Caymanians were unable to rescue.

Source: http://www.atlasobscura.com/places/wreck-of-the-ten-sail

I had their attention and my hopes were that I had added some humanization to the problem we were about to work on. While the mathematics were the same, I had changed the story setting as I have been to the Caymans. After class I was surprised by how many students wanted to come tell me their own stories about the trips to Caribbean or to share facts about the Cayman crocodile.

In the question, I wanted to continue developing both the conceptual understanding of linear functions as well as the skills of writing equations to represent scenarios. We had worked on a similar question in class,

although the in-class question was not as complex. This PoW could be solved using a variety of strategies: table of values, substitution, and even graphing. The majority of students, even the students who were unable to finish the question, recognized that the taxi's bill must equal \$15.00. From that point on, there was an equal quantity of students who chose to substitution to identify the missing valeuys and students who chose to use a table of values to arrive at the solution. One student attempted to represent the 3 vehicles on one graph. Regardless which strategy they used, most students were able to identify the equations to represent each vehicle.

Problem of the Week #2 THE "BIG IDEA" BEHIND IT

Analyzing simulations and data allows us to notice trends and relationships.

(BC Ministry of Education, 2015-b)

From Here to There

I would love to visit the Grand Cayman (the largest island in the Cayman Islands). If I do go, after I have landed at the airport I will need to get to my hotel.



I have investigated my ground transportation options, and found the following:

- A bus runs from the airport and stops at all hotels for a \$15.00 fee.
- Taxis in Grand Cayman charge an initial fee of \$2.00 for the first 1/4 mile or fraction thereof plus \$0.25 for each additional 1/4 mile or fraction thereof.
- A motorcycle shuttle (with sidecar for luggage) charges an initial fee of \$3.00 plus an additional 1 cent per second.

For each transportation option, pick the graph below that you think best illustrates the cost of riding it to my hotel. For each graph you choose, **explain why you think it's the best fit and include the units that you would put on each axis.**



Under what conditions, would all three options cost the same to get from the airport to the hotel? Explain your thinking.

PoW#2 - Wee	PoW#2 - Week 1: The Initial Submission				
Number of submissions:	Number of student that scored 10/10:	Number of students that scored 1/10 but were very close and on the right track:	Number of students that scored 1/10 and had significant errors:	Number of students that did not submit:	
26 out of 29	10	6	10	3	
There has been an increase from 24 to 26 students - a 7% increase.in the quantity of students who submitted this assignment on the first week	This means that 10 the class were either c track. It is similar to P less overall (17 stud quantity of students t the first submission w and PoW #2, howev received 10/10 in	5 students, or 55% of orrect or on the right oW #1 but is slightly ents). Ironically the hat scored 10/10 on vas equal on PoW #1 ver, only 5 students both situations.	This means that 13 the class did not demon understanding of this p slightly higher than Po' problem was more cha required a combination and logical reasoning in	students, or 41% of nstrate conceptual problem. This is W#1, however, the llenging and n of multiple skills n order to solve it.	

PoW#2 -	Week 2:	The 2 nd	submission
100074	- WUUM 2.	Inc 2	Submission

Number of possible submissions 19	Number of students who resubmitted for their 2 nd time 10 out of 16	Number of students who submitted for their 1 st time 2 out of 3
Ironically this is the exact same as the PoW#1	This means that 12 out of 19 students too submit their work. This is simil	ok advantage of this second opportunity to ar to PoW #1 (13 out of 19).

PoW #1 – Focus Group Results

All 4 students made the initial submission, however,	
they all had significant errors in their process. One of	
the students used a formula that has not been taught	Only 1 out the 4 students took advantage of the
yet but did not understand it and did not apply it	resubmission. It was a different student than the
correctly. When asked about it, she stated that a sibling	one who resubmitted on PoW #1
had helped her but that neither of them knew what to	
do with the formula.	

FreshGrade Student Reflections

Student A	Student B	Student C	Student D
that she worked with also did not resubmit.	was not correct and did not submit	assignment and did not resubmit	resubmitted and made corrections
Note: This student did not resubmit. The peer	well. Note: This student	Note: This student only completed half of the	thinks <i>Note: this student</i>
together so it was easier to do, it's a kinda tricky problem if it's not analyzed properly	answers for them. I do think I would've been able to show a lot more of my work, but I had a lot of other homework to do as	thought i was wrong at first but last semester i did a lot of graphing in science and am now good at it.	for each question and graph and we both agreed on the answers it did give me that security that what I think is he what he
me and a friend worked on this one	question at first. I had my older sister help me with this assignment, and she helped me find the	Really easy i finished it in a couple minutes i	I did collaborate with people like Ishwar we both had the same idea
	This PoW was challenging for me, because not only was it a little time consuming, but also because I had a hard time figuring out the		it was 50 50 it was challenging and time consuming at the same time

PoW # 2 – Final Data	_
Mean Score /10	7.2
Standard Deviation	3.9
Percent of students who submitted on the first week.	90%
Percent of students who took advantage of 2 nd opportunity to resubmit	63%
Percent of students with at least 1 submission	97%

These results are almost identical to PoW #1. The questions were similar in style, yet PoW#2 was certainly more challenging

This is a 5% drop from PoW #1

Chapter 14: PoW #3 – Changing direction – cruise ships

This week I wanted to try "Noah's Ark" again – the same one I had tried in the preliminary trial during from the first semester. I changed the name to "Cruise Vacation", used different animals, rearranged the levels, and made minor adjustments to the wording. I did this to avoid students looking online for the solution. So far, this is the only PoW that I can readily find the solution for online, that is, unless you pay for the Math Forum subscription, but that does not mean that the solutions are not available somewhere online.

In this problem there are numerous ways to solve it and while the big idea behind it is the same as the first PoW, it has a very different feel to it. At the end of the action research I surveyed the students and many students claimed that this particular question, even when compared to ones we had done in class, was their favourite because it "was really challenging but fun to do". Algebraic reasoning must be used and the solution can be connected to polynomial expressions.

Problem of the Week #3 THE "BIG IDEA" BEHIND IT

Operations between polynomial expressions are connected and allow us to make meaning through abstract thinking.

(BC Ministry of Education, 2015-b)

This question is far less wordy than the first two questions. Ironically, I noticed in the first semester that it is the high achieving students who make this the most complicated by using a multitude of equations, however, it can also be solved using a balancing strategy of left versus right sides of boat - while temporarily ignoring levels.



CRUISE VACATION

A bunch of animals are going on a cruise. The captain needs the boat to be balanced when the animals are resting in their rooms. The ship is divided down the middle and each level of the boat has been balanced so that the weight of the animals on the left is exactly the same as the animals on the right. The only level that has space left and has not yet been balanced is the fourth level. How many seals should be in place of the question mark so that they (and the turtle) weigh exactly the same as the 6 penguins?



PoW#3 - Week 1: The Initial Submission Number of students that scored 1/10Number of students ... but were very that scored 1/10Number of Number of students Number of student close and on the and had significant submissions: that scored 10/10: right track: errors: that did not submit: 14 2 0 13 16 out of 29 This is a drastic Although less students submitted, the decrease from number of students that initially scored the week prior. 10/10 increased by 14%. Ten less students

Ten less students	
submitted this	
week. If find	
myself	
wondering if I	
have	
unintentionally	
set up students	
to procrastinate	
since they know	
that they can	
hand in and still	
receive 10/10 on	
the second week.	
I do not assign	
very much	
homework in	
this class and am	
discouraged by	
this lack of	
submissions.	

PoW#3 - Week 2: The 2nd submission

Number of possible submissions 15	Number of students who resubmitted for their 2 nd time 2 out of 2	Number of students who submitted for their 1 st time 12 out of 13
	While the initial submission was very low – o submission increased the total submission to not hand in on the first week, but I am please week. 14 out of 15 took advantage of the sec that did not submit is not part of the focus gr completing assignments and has also had une of the official focus group, now that I am away eye on this student.	nly 16 students, the second week 26 students. I am not sure why so many did 26 dthat so many submitted on the second 27 ond week resubmission. The one student 28 oup but has been inconsistent in 28 excused absences. Although he is not part 29 are of this trend, I will be keeping a closer

PoW #3 – Focus Group Results

Only 1 out of the 4 submitted on the first week and she received 10/10.

All three remaining students took advantage of the resubmission and scored 10/10

FreshGrade Student Reflections

To be honest, I feel like this was actually the most time consuming POW for me, because it took me really long to find out the solution. Not necessarily the final answer per se, but rather, the entire thought process. It was difficult for me, but once I found out the equations and such, it was pretty much just tidying up the mess and things.

No reflection completed

Note: This student submitted on week 2 and scored /10

Student A

Note: This student submitted on week 1 and scored her first 10/10.

Student B

It was a little hard at first but when i started at it it became easier as i tried new angles

It only took me about 10 minutes but i did have fun with this one

Note: This student submitted on week 2 and scored _/10

Student C

POW- this POW seemed pretty simple to me since the image was so simple since I've seen something similar to this on social media before but when I started or tried to do solve the problem it definitely took time to solve. I knew from the the start that you would have to use the image to see what equals to the 6 penguins. I worked with a friend to see what two minds can do to solve it

Note: This student submitted on week 2 and scored _____. His friend submitted on week 1 and scored 10/10.

Student D

PoW # 3 – Final Data	
Mean Score /10	9.2
Standard Deviation	2.4
Percent of students who submitted on the first week.	55%
Percent of students who had either 0 or 1 and took advantage of 2 nd opportunity to resubmit	93%
Percent of students with at least 1 submission	97%

Chapter 15: My problem - feeling scattered

In the middle of my action research I felt myself feeling a bit scattered and that I'd taken on too much. Problem solving? That's a huge topic! With the problems of the week, I found myself questioning the purpose of my research. Was the goal to find a way to increase completion rates of assigned work? Did I want students to complete the problems in order to complement themes covered in class or did I just want them to work through a tough problem and enjoy the sense of satisfaction that comes with that? If that was the case, were some of these questions too easy? Were some of them too hard? Was my goal to increase 'grit' in my students by providing these challenging problems of the week? Did struggling learners actually feel more discouraged?

I continue to debate the aim of my research focus. Was I trying to extend the collaborative culture of our class to after school work time? I had hoped that the mathematical conversation would carry over to the dinner table, but if it did, was that just an added bonus? Was the use of FreshGrade increasing their accountability and completion rates? I hoped it was inspiring them, but was spending 30 minutes once a week uploading to FreshGrade and having students write reflections on their curricular competencies a waste of valuable class time? Or did the students see value in this online portfolio? On that note, I wondered if parents even looked at their child's work because very few had left any comments. On the other hand, parental support, while wonderful to have, wasn't my primary goal in using FreshGrade. I wanted the students to use it for themselves, as way of documenting a collection of problems that they had worked very hard on. Did the students have any insights while doing reflections or were their write-ups contrived? By expecting students to work on it outside of class time, was I actually encouraging spontaneous concepts to develop (Vygotsky, 1934/2012) or was I developing systemic understanding because I tried to choose problems that would complement either what we had learned or that the students had learned in the past? When I added more supporting details to add to the story, did that add intrigue to the students experience or was it fluffy hook?

On the other hand, so much of what I had questioned was focused on the experience and learning of the student. Was this something bigger than that? Honestly, I already feel that I am a very effective teacher. I

constantly reflect on successes and failures I've had in my class. When I get the sense that students don't understand, I often modify my own teaching or try different activities to see what I can do to improve. I am an effective teacher, but I also recognize that I am growing and still learning. I enjoy being challenged and challenging my own professional growth is something that I enjoy. In the process of this action research I have become much more attentive to the details of the day and am more aware of the subtleties in the activity choices that I am making. Perhaps this was truly the main goal of this research.

While working after school one day, marking my grade 12 assessments on cubic functions, I felt inspired to create a graph to represent, generally speaking, my enthusiasm and passion for my career since I began teaching. It's not precise with regards to the regular ups and downs, but I feel it gives the gist of the ebb and flow that does happen. The dashed line represents a state of neutrality. The dotted line represents my time spent acquiring my Masters of Education. I am beginning to recognize and appreciate more and more that this action research is actually more for me than for my students.



Chapter 16: My solution - Make it worse

I was feeling scattered with goals unclear. And then I heard the following words from a recent memory:

"Sometimes you need to make a problem more complicated in order to make it easier"

Richard Hoshino, BCAMT conference speaker, 2016

This is interesting...sometimes you need to make a problem more complicated in order to make it easier. I can do this.

So I decided to immerse our entire class in problem solving. We would learn the required content of the course, but we were going to do it via problem solving. My goal was to work on big, challenging, multi-step, outside of the box type of questions almost every single day. Some would have more information than needed, some would have enough information but it wouldn't be instantly obvious. Some questions would have a variety of possible strategies that could be used, some would have step-by-step instructions that needed to be followed precisely. I decided I wanted us to eat, drink, and breathe problem solving every day for one hour and 19 minutes while I had these students in my grasp.

The first thing I brought in was a bright orange problem that I had been inspired by from The Math Forum website. It also connected to a four fours order of operations challenge we had done in class (Four fours puzzle, 2016). On the orange poster board I had created 100 entry spaces to represent sums of 1 to 100. I opened this challenge to all classes and told them that *if* they could solve it, then all classes would get a free block designated only for games, puzzles, and delicious treats. They must use all the digits of 2,0,1,and 7 but they could only use them once. Together the 4 digits, combined with any operations, needed to equal the sums of one to one hundred. I have no idea if this can even be done. Students immediately began working on it, not only during class, but also before and after school as well.

10		2017	A CONTRACTOR OF THE OWNER OF THE	1
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L Y L	23	52	77	6
	28	53	78	844
	29	54	79	147
14 S	50	55	80	122
6	31	56	81	310
7	32	57	58	1018
8	33	58	85	11.1
0 6 9	54	51	84	
10	35	60	85	6.1
II C	36	61		0.0
12	37	62	87	nd
11/0 15	38	63	88	20
2/3/14	39	64	89	8
15	40	65	90	E sea
16	41	66	91	1
FI 13	42	63	92	6
18	13	68	93	
	44	69	94	200
270	45	70	95	22
2 21	46	71	96	20
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65	50	142		

The next step was to be selective in how I chose to use the time during our class. I never really have been a 'classic' math teacher. I don't enjoy doing notes followed by silent seat work. I do follow this approach, but it is not something I regularly do. There are times to work in silence when you need to silently think your way through something, but I also quite welcome the noisy, but still engaged, classroom environment. I had seen a lot of success with this particular group of students when they worked in partners, even if I chose the partners for them, and elected to make more use of the giant whiteboards that fit nicely over the entire desk space.

To get the problems, I bypassed all the skill builders from my resources and went right to the "application" type of questions. I used Marion Small's approach to differentiate instruction to reach more students by using the strategy of turning closed ended questions into open ended questions that could be solved in a variety of ways so that all so that I could "teach to the big ideas" (Small & Lin, 2010).

In the next class, I presented a problem that incorporated polynomials and the area of rectangles – I called it the kite question. I noticed Student A multiplying polynomials in a manner I had not taught. I asked her where she had learned this approach and her response was that she had gone online and watched many videos to help her understand better. I was quite impressed that she had taken this initiative, however, I later noticed that she was making errors when she was multiplying binomial with trinomials. Together, her and her partner, student B, tried to identify the error but they were unable to find it. I helped them figure it out and then solved the same question using an area model so that they could compare these two strategies. I told her that she was welcome to use either one but she needed to be conscientious of the step in which she was making errors.

At one point I returned to Students A and B. They now had the correct answer but were not able to explain why they had performed an operation of dividing by 2. At this point I was not sure if they had overheard the answer and recognized that if they manipulated their value by dividing by 2 then it would equal the final answer or if they in fact recognized that the area of inner diamond was actaully half of the rectangle's area. This reminded me of Vygotsky who stated that in the critical period of adolescence, the student may be able to use a concept correctly in a concrete situation but will find it "strangely difficult to express that concept in words" (Vygotsky, 1934/2012, p. 150).



A similar question to ours. Source: https://www.slideshare.net/emamkhan923/math-quiz-final-round-with-answer

Another question we worked on during class had some excellent results. It seemed that every single student was actively engaged for this question. In this question, they needed to apply the multiple strategies in their application of simplifying polynomials to determine the amount left over after two strips were removed from the edge of a rectangle.

This work was done by two students working together. There was a lot of dialogue as they made their plan and talked their way through their solution





"Pretty easy. I finished earlier than everyone and went around helping."

Student C, proud of being able to solve the assigned in-class problem. He took the initiative to post this picture on his FreshGrade account so that he could share with it with his family



"Everyone else in the room is bored of this question and we're still working on it!"

Said with a smile by two students who spent 45 minutes on this question. They had solved it within the first 15 minutes but spent the next 30 minutes testing out two other ways to solve it. Afterwards, they presented their solutions to the class and they were very proud! This feeling of being puzzled by new questions that arose even after they had solved the question is a state I want all my students to feel. This puzzlement is truly part of the joy of mathematics and Zazkis and Zazkis (2014) believe that with an appropriate pedagogical approach, this puzzlement can be "contagious" to students.

Descriptions and samples of more of the problems we did can be found in the chapter "PoW#4 – The build up" as well as in the chapter "More in-class problems".

Another way that I decided to make my research more complicated was inspired by our school librarian. He had emailed out an article to all staff several months ago but it sat unread on my "to do" pile until one day, while cleaning, I stumbled across this article all about "The Mystery of Motivation" (Drevitch, 2017). In this article, the author discusses how commonly used incentives backfire and proposes alternatives that would get people to do the "right" thing (Drevitch, 2017). One idea that caught my attention was related to social pressure. If people's behaviours are observable to others, it can create social pressure to make better choices. It seemed that the completion rate was consistently higher than my trial run and I wondered if the 30 minutes of class time every Thursday in which the students sharing their completed PoW's with their peers, and documented their progress online via FreshGrade was actually helping to motivate the completion of the PoW's. By informal interviews with the students, using FreshGrade at the same time as their peers appeared to be a helping factor in pressuring them to get the task completed.

As I was feeling inspired by the insights of this article on motivation, an idea evolved regarding the reporting of their scores. The article discussed various incentives used to reduce the use of plastic bags for shopping. It seems that if shoppers are given five cents credit or a deduction off of their grocery bill, it has very little effect. However, if the shoppers are charged five cents for every bag they use, many more shoppers bring cloth bags with them (Drevitch, 2017). Even with something as small as a nickel, we feel more pain when we lose it than we feel pleasure in gaining it – this idea is referred to as "loss illusion" (Drevitch, 2017). I transferred all scores to a spreadsheet, and instead of tallying up how much they had earned, up to a possible 30 points so far, I deducted whatever points they had missed off "100" and plotted the results on a bar graph. I was hoping to tap into the emotional side of their motivation and take advantage of this "loss illusion".



I posted this graph for all students to see and their reaction was not what I expected. I explained what I had done, students looked at it, shrugged their shoulders, and walked away. Perhaps I should have shown

how much they had dropped from a potential 30 points instead. Perhaps it would have been more motivating if I'd started it at the beginning. Perhaps I should start fresh with each PoW and show how their score had dropped from 10 by not handing it in. Regardless, the whole thing was quite time consuming, and even it did motivate students, I think that there are better more efficient strategies.

Chapter 17: PoW #4 – The build up

As we are now further into our course, the set up for this problem has been much more elaborate than the others, however, we are still focused on adding depth to our conceptual understanding. I am not sure if the Problems of the Week that I have selected so far have truly complemented my layout of the course, and will likely try different problems the next time around. This PoW, however, is different. It very much complements what we are doing.

I teach Math 10 using a spiraled approach. The course has four major themes: Measurement, Algebra & Number, Relations and Functions, and Systems of Equations – in other words, M.A.R.S. When redesigning my layout last year I was excited to notice this acronym and set up the course so that we literally visited M.A.R.S. every week. I am still refining the sequence of sub-topics. Adding to

Building up to Problem of the Week #4 THE "BIG IDEA" BEHIND IT

Analyzing simulations and data allows us to notice trends and relationships.

(BC Ministry of Education, 2015-b)

the workload is the BC new curriculum that I am also incorporating. While I assess based on individual skills used within each area, I have come to recognize the value and lasting significance in focusing on building the students conceptual understanding first. I feel that by teaching the course in this manner, it will compliment Vygotsky's belief that the initial focus should be on the abstract concept and then, through carefully planned mediation of complex activity, the students will develop full systemic understanding (Vygotsky, 1934/2012).

PoW #4 is connected to the measurement, or trigonometry, part of our course, and we have just begun. Over the course of the lessons building up to the distribution of PoW #4, students have made comments about this course that I hadn't noticed before. They commented that almost everything we've done is based around rectangles and they're right! We've covered polynomials and represented their addition and subtraction via the perimeter of a rectangle. We multiplied and divided polynomials using an area model, which is just another rectangle. We've used an online graphing calculator to sketch lines, practice restricting domain and range, all to build... rectangles. And now we've chopped a rectangle in half and are learning about triangles.

Part of trigonometry is determining the value of missing side lengths. Pythagorean's theorem is a relationship that enables us to accurately identify missing values and is something that they have all learned in grade 8. However, I suspect that many of them were just blindly following a formula and not truly understanding the relationship. So, working in partners on giant whiteboards, I presented them with the following set of statements (Hunter, 2014):



One side of a right triangle is 3metres long.

Another side is 4 metres.

The last side must be 5metres long.

Is this ..."Always, Sometimes, or Never" True?

Students worked on the giant whiteboards in groups of two and I had rearranged students to be with new

partners. I intentionally sat Student A with Student B and for the first time in this course, they were both

noticable engaged in converstation about what to do. Student A struggled with setting up the formula and then with manipulating to solve for "c", and Student B explained how to do it both with and without a calculator.

I was surprised at how many students had actually forgotten Pythagorean's formula; there were approximately eight students that either wrote the formula wrong, or manipulated it incorrectly. From here we had a class discussion about Pythagoreans theorem, why it really means, the formula, how to label right triangles, as well as what "counter examples" were. After we reviewed this, for many students, this problem seemed really easy to start but then they got into heated discussions. From their debates arose several groups who identifity counterexamples to prove that the "always" just was not true. In our class sharing, student C was eager to share why the hypotenuse could be 4 meters but not 3 metres, therefore, elimiating one of the two suggested counter examples.

Another question we worked on in parterns involved identifying the length from one corner of a rectangular prism to the diagonal corner, given only the lengths of the edges of the prism. This question applies Pythagorean's theorem, but connects to a future topic involving rise and run, or slope.



Image source: http://math.tutorvista.com

I found it fascinating that in this rectangular prism question, Student A and Student B solved it differently than everyone else, however, their sketch was actually easier to visualize. I have used this question for about a year now and never thought of it the way that they did. I've had students work on it independently and none of them saw it this way either. I expressed I was very impressed and requested student A and student B present their work their work to the class and they seemed proud to do so.



How most students, and myself, visualized the problem.



How Student A collaborating with Student B visualized the problem.

On a side note, this class seemed to solve this question quicker than any group I've had before. I feel that this happened because I have set the culture to be one of collaboration and problem solving and this class is embracing this culture. Additionally, I will often show "mistakes" from group work since we learn through our mistakes, however, I didn't want to discourage these two students, who had seen the question differently from the rest of us, and helped them to correct errors in their calculations before I had them present their solution to the class.

Chapter 18: PoW #4 – question and results

I really wanted to avoid misconceptions regarding right triangles and hoped that the in-class problems concerning right triangles had been an effective use of mediation in order to avoid spontaneous

misconceptions (Vygotsky, 1934/2012). I had hoped to bring in some homemade quilts that were gifts to my children, but unfortunately did not. When I use this problem again, which I will be sure to do, I will bring them in to help inspire conversations and encourage students to feel connected to the question.

A Quilting Project

Problem of the Week #4 THE "BIG IDEA" BEHIND IT Proportional comparisons can be made among right triangles, using

trigonometry.

(BC Ministry of Education, 2015-b)

The word quilt comes from the Latin <u>culcits</u> meaning a stuffed sack, but it came into the English language from the French word <u>cuilte</u>.

The origins of quilting remain unknown, but historians do know that quilting, piecing, and applique were used for clothing and furnishings in diverse parts of the world in early times. The earliest known quilted garment is on the carved ivory figure of a Pharaoh of the Egyptian First Dynasty about 3400 B.C. In 1924 archaeologists discovered a quilted floor covering in Mongolia. They estimated that it dates from somewhere between the first century B.C. to the second century A.D. There are also numerous references to quilts in literature and also inventories of estates.

Source: https://www.emporia.edu/cgps/tales/quilte~1.html

I would like to make a quilt using the "Ocean Waves" pattern, because I really like how the right triangles make a spiral pattern.

The quilt is made up of 36 big squares, arranged in a 6 x 6 array. Each of the 36 squares is made up of nested, smaller squares. The quilt also has a border that is six inches wide.



I would construct each of the 36 big squares by starting with the small inner square (dark red in the picture) and sewing isosceles right triangles to each side of the square, so that the hypotenuse of the triangle is sewn to the edge of the square. This will make a larger square.

Then I would sew another four isosceles right triangles around the new larger square, again sewing the hypotenuse of each triangle to the edge of the square.

For each of the 36 squares, I'll repeat the process of sewing four triangles around the edges of a square six times.

I want the whole quilt to be 7 feet by 7 feet.

Note: there are 12inches in a foot



What should the dimensions be of the dark red center squares?

PoW#4 - Week 1	The Initial Submission	on		
Number of submissions:	Number of student that scored 10/10:	Number of students that scored 1/10 but were very close and on the right track:	Number of sudents that scored 1/10 and had significant errors:	Number of students that did not submit:
18 out of 29	16	2	0	11
	Note: several students forgot to account for the border, but I chose to give them full credit. Next time I will be sure to stress this aspect of the numbers when I initially distribute the question.			
PoW#4 - Week 2	: The 2 nd submission			
Number of possible submissions	Number of students their 2 ^r	who resubmitted for ^{id} time	Number of student their 1	s who submitted for I st time
13	1 out	of 2	6 ou	t of 11
	For this PoW, 5 study suspect that the two we both submission date pictures of their work	ents never did submit veeks spread out over s s, I gave very little cl to FreshGrade.	. This is the lowest co spring break influenced ass time to collaborate	mpletion so far and I I this. Additionally, on e while they uploaded

PoW #4 – Focus Group Results

3 out of the 4 submitted on the first week. Two of these received 10/10 and the third was very close but made errors in last steps and received 1/10.

All four students ended up submitting this PoW at POW- I thought this POW was hard because there were many ways you could have engaged this question and get a different answer but be close to all the others I worked with pardeep and ishwar to answer the following question

least once with scores of 1, 10, 10, and 10

FreshGrade Student Reflections

"In this PoW, we demonstrated understanding and solving because we used phytagorean theory to solve the problem. i collaborated with two friends at	I used the pythagorean theory to solve the problems. I also collaborated with two friends during lunchtime to finish the		hard because there were many ways you could have engaged this question and get a different answer but be close to all the others I worked with two friends to answer the following question
lunch, to finish it. the way this problem of the week was solved is connected to the previous lessons we had."	PoW together. For each triangle, we had to use the same formula for each one to solve it altogether.	No comment/reflection left on Freshgrade	Note: This student submitted on week 1 and scored 1/10. He had an answer very close to correct answer, however, there were steps in his
Note: This student submitted on first week and scored 10 /10. In	Note: This student submitted on week 1 and also scored 10/10. She worked on the in-class	Note: This student did not submit on week 1.	solution that seemed random and he was not able to explain why he, for example, divided by 6
class, when we had worked on a similar problem she was able to figure it out and said "it feels good to understand".	similar problem with Student A and agreed that once they were able to figure it out that it "felt good".	He did submit on week 2 and received a score of 10/10	<i>He did not resubmit and claimed to "have forgotten" about the assignment</i>
Student A	Student B	Student C	Student D

PoW # 4 – Final Data	
Mean Score /10	7.5
Standard Deviation	4.1
Percent of students who submitted on the first week.	62%
Percent of students who had either 0 or 1 and took advantage of 2 nd opportunity to resubmit	50%
Percent of students with at least 1 submission	83%

Chapter 19: More in class problems

Since beginning my studies into Imaginative Education, I have shifted the way that I teach trigonometric ratios and my lessons are still evolving. Last semester I focused on understanding the significance of the ratio of opposite versus adjacent side lengths prior to introducing the tangent function capability of our calculators.

This semester, with problem solving as the focus of the majority of our classes, we began by reviewing how to draw angles using a protractor. We then drew right triangles of different sizes with a 30 degree corner, measured the opposite ide lengths and the adjacent side lengths. We then compared these lengths as a ratio of *Opposite length* and used the front board to tally up our results in a chart All of the 30 degree triangles had ratios of approximately 0.6. This led to a lesson regarding similar triangles and proportionality. We then sketched a 40 degree triangle and created another chart. Again, the majority of the values were the same, a value of approximately 0.8. By doing this as a class activity, I was very quickly able to help anyone who was either having issues with measurement or had accidently compared adjacent to opposite. When asked to prove to me that their triangle was a right triangle with a 30 degree corner, Students A and B were unable to use the protractor and had estimated their drawing by looking at triangles from students around them. I was able to help them one on one and by the end of the block both were very proficient at effectively using the protractor. If I had not been doing my action research with a focus on these two students, I am not sure I would have noticed their struggles as their estimated sketch was actually fairly accurate. This class activity took an entire block.

The next day we summarized our results and made a prediction that the ratio would always be less than 1. Students worked in partners to decide if this was "always, sometimes, or never true". All groups were able to provide at least one counterexample, or anomaly, to our conclusion.

Results so far ... trom following hy outhoris: 8.0 % othat 010 ratio Tati alwar Always ratio ≈ 1.8

From here I asked if we could make some generalizations about the target angle based on the ratio. Students worked for approximately 10 minutes and had some very animated discussions about their predictions. Student C was working with another student and they were particularly excited to share their results. Maybe it was because we have been relating everything back to rectangles, but their explanation showed a clarity in understanding that I have not seen from students this early in our studies of trigonometry. These two students volunteered to share their reasoning with the class.

Student C: "it feels like there's a rectangle around a 40 degree triangle and that ratio would be less than 1. Then there's a square and that ratio is one because a square has all sides the same. Then the last triangle, with the 60 degree corner, has a ratio greater than 1 and it can be drawn inside another rectangle but this rectangle is up and down because the opposite side length was bigger than the adjacent."

Teacher: "Can you make any conclusions or predictions about the target angle of the triangle with the ratio of one?"

Student C: "We think that that that triangle, with a square around it, would have an angle of about 50 degrees because that is right in the middle of 40 and 60"

There was a bit of a misconception here regarding the 50 degree prediction, however, they did recognize that it had to be between 40 and 60 degrees. I was impressed with their visual connection of triangle properties v versus the properties of rectangles. From here, we continued the discussion and another student pointed out that a square has 90 degree corners so if a line is drawn from one corner to the next, thus becoming the hypotenuse of the right triangle, then the corner is split into two equal angles of 45 degrees. Therefore, a right triangle with a 45 degree corner will have an $\frac{Opposite}{Adjacent}$ ratio of 1. Another student commented that this would always be the case, no matter how big the square was. After class was over, Student C and his partner came up to me and asked if I could upload a picture onto their FreshGrade accounts of what they had presented because they were "really, really proud" and wanted to share it with their parents. I had already erased the board and instead took a picture at the start of next class which reflected the conclusions that they had made.

Mesterdau

From here I decided to continue developing this abstract reasoning, and made the following predictions:

My next idea... If I compare the site lengths of orp & hyp I will get the some results. Test my idea et provide built examples counterexamples to reflect "alweys, somethings, or never true"

All students were actively engaged immediately. One student comment "I bet it's sometimes but we have to figure out examples to prove this"! As a group, they quickly summarized that these ratios are always less than one and never equal to or greater than one. Two students made an observation that I was not expecting: that for the case of an isosceles triangle, which is half of a square, whatever the $\frac{Opposite}{Hypotenuse}$ ratio is, it will be equivalent to the $\frac{Adjacent}{Hypotenuse}$ is. This is true.

Controly be L 1 out the ratio will never equal "1" be

When the students made references to the properties of rectangles while studying triangles, it made me reflect that many of these students are on the verge of philosophic understanding. They are focusing on the connections and are making links between ideas. They are experiencing interactions between their general schemes and new knowledge (Egan K. , 1997) and these moments in class remind me that I need to continue to provide experiences that stimulate this philosophic understanding.

Chapter 20: PoW Reflections - PoW #1 through #4

	PoW #1	PoW #2	PoW #3	PoW #4
Mean Score /10	7.3	7.2	9.2	7.5
Standard Deviation	3.8	3.9	2.4	4.1
Percent of students who submitted on the first week.	83%	90%	55%	62%
Percent of all students with at least 1 submission	100%	97%	97%	83%

After completing the first four "problems of the week" for this course, I tallied up the results to date. I will continue to keep track of the results as I find the data presented in the following manner interesting.

I find it interesting that PoW #3 had the highest completion as well as highest mean score as it was I think the most challenging. At the end of my research period, I asked students which question, out of all questions we had done in the course both in and outside of class, was their favorite. Most students commented that PoW#3 was their favorite. Even the student whose learning support teacher complained that he had spent too much time on it (he had spent approximately 3-4 hours on it), commented that it was his favorite to work on.

I strongly encourage my students to sketch out all problems, however, in doing research for this project I have learned that if drawing are done as "schematic" – in other words with values and proportions included as compared to "non-schematic" in which drawings are presented without any numeric data, there is a higher level of spatial understanding and problem solving performance (Edens & Potter, 2007). These problems of the week have provided many opportunities for students to explore various ways of organizing and manipulating their data, in other words they have applied 'the literate eye' (IERG, n.d.).

Student Samples from PoW#1 that reflect different ways of organizing their data:

The company should build the new boltery store bar 4, and here's why: Store B a L are the dosest to the middle this helps because instead of having to go back to stare A or D everytime the track only has to go to store C a. B. Things Gtimes Let's Compare: If it was at store A -B=8+8=16×7=112 -C=14+14=28×6=161 -D=25+25=50×9=45 Total miles: 730 miles 8 filmes 7 times 9 times If at store C: 8 miles 86 miles C Il miles C D= ||+||=22 ×9= 198 3=6+6=12×7=84 =1414=28×8=22 ame amount Total miles: 506 miles milez or epich 8times 6times 9 times location it could either If at store B A 8 miles Blomiles " miles D = B + 8 = 16 × 8 = 18 = 6 + 6 = 12 × 6 = 12 = 17 + 17 = 31 × 9 = 306 Total miles: 506 miles wolte 506 miles

PoW#1 - This student solved PoW#1 using cases.

Olives Things 6 times Let's check store D just in case: 1 ABrild Bland "mikes 11 + 11 = 22 × 8 = 176 17 + 17 = 341 × 7 = 238 25 + 25 = 50 × 8 = 400 Total miles = 814 miles miles The numbers show that store (and/or B waid be lest suiting for the bakery, as they waid make the traveling distance for the bakery posts as law as possible. The reason for this being both stores are close to the middle which allows them to not have to travel from are end of the road all the way to the other, like stores. A and D waid have to The menefit of being close to ar in the middle is that you are as close you can be to the other stores. Therefore the baking company would have the lowest weekly distance to a battery was implemented into either store B or store C. Store C.



PoW#1 - This student's first submission for PoW#1 using a diagram but it only showed one case. On the re-submission, he switched to using another approach in which he algebraically showed the four cases.

	-fait
A-B=6×8=48milu	
$A-C=7\pi 4=98m l/W$	
$A - D = 9 \times 2S = 2ZSmilw$	
E@A = 371.21 = 742 milu	
	1
B-A= 8+0= 64M/W	
B-C = 7/6 = 42m/hu	
$B-D = 0 \times 17 = 153 \text{ m} / 1$	
ECB = 259/2 = 518 mi/N	-
010	
C-A= 8×14=112milu	
C-B = Cex(0) = 36mi/m	
$C = 9 \times 11 = 99 \times 100$	
EOCE 247milw	
U-A= 8x25=200 milu	133-
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Student Samples from PoW#2 that reflect different ways of organizing their data:

Best Filting Graphs The Busing graph for the bus fee would be the best filting graph for the bus fee would be graph D. I picked graph D because, no matter how Graph D for it is to the hotel, there is just a that out payment of fifteen dollars. No more and no less. This is represents well on graph D as the line is straight and does not go up or down, indicating there is no change in cost. 10+ The Taxis The test filling graph for the taxis that the graph C. I think this because this graph is the only graph that traph c doesn't increase gradually, just like the payments of the taxi. For example, if you payed the starting fee of \$200, then travelled a '4 mile but decided to travel a '4 mile more the payment goes up instantly to \$225 pich if you and end up traveling the whole rity up 34, 34, 44, 54, 64, 74, mile the payments increase instantly, the Distance limits represented in the graph (U, not increase gradually. , Laxi: (15) Pravo gradually.

PoW#2 - This student has made strong conclusions regarding linear functions

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ap initial price is set, and it gradually raises as the distance (0.25mi) rolls of The cost raises every quarter mile.
Motorescie
The motoraycle option is best represented by the Graph & bleause the price stats relatively low then is constructly rising for all the distance travelled. This

PoW#2 - This student used a combination of tables, equations, and explanations to solve this question

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(Cost = \$15.00)
Conditions to have same cost:
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Texi: Mile Cost(\$) Othe hotel would need to 0.25 2 be 3.25mi away from the 0.50 2.25 alipoit to pay \$15. 0.75 2.50 (1.00 2.75) (3.25) 15
Motorgale: 515 = initial prace(55) + 1200 seconds of travel (20 mins).
12003 = \$12 131
® The motorcycle would have to take a route that takes 20mics to travel to the potel (3:25 mi away).
The statements made in point @ + @ would have to be true in order for each mode of transportation to cost the same.

Student Samples from PoW#3 that reflect different ways of organizing their data:

CRUISE VACATION A bunch of animals are going on a cruise. The captain needs the boat to be balanced when the animals are resting in their rooms. The ship is divided down the middle and each level of the boat has been balanced so that the weight of the animals on the left is exactly the same as the animals on the right. The only level that has space left and has not yet been balanced is the fourth level. How many seals should be in place of the question mark so that they (and the turtle) weigh exactly the same as the 6 penguins? 313

PoW#3 - This student used the strategy of assigning weights to each animal based on logical reasoning.

Applem of the user	
level 1 Meight = 3 pounds on each side level 2 weight = 6 pounds on each side level 3 weight = 10 pounds on each side level 4 weight = 6 pounds on each side level 5 weight = 12 pounds on each side.	
So since 3 penguins is equivalent to 1 turtle you would need a number of seals to be exquirvalent to the other 3 turtles you would need 6 seals. Jeal = .5.	


PoW#3 - This student used algebraic expressions and substitution to represent each animal.

	Seal=s 6p+2d=2s+1h+1+ hipm=n 6p+2d=2s+6p+3p
	perguin = p $Gp + 2d = 2s + 9p 1 - 6p2d = 2s + 3p$
	turte = t
	1 Mippo = 2 turtles/6 perguins 1 turtle = 3 perguins 1 deer = 1 seal + 1.5 perguins
	penguin=298als
	$\begin{array}{c} 4+=2d+1h+45\\ 12p=25+3p+6p+45\\ 12p=65+9p+9p\\ 3p=65 \end{array}$
	$\frac{3p=65}{3} = \frac{p=28}{25 \times 3} = 65 = 3p$
*	Since liturate is equal to 3 penduins and you need a number of stats to be equivalent to the other 3 turtles, you would need 6 seals.
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Student Samples from PoW#4 that reflect the literate eye:



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	each side of third square; half = <u>7</u> = 3.5cm 2	cach side of fifth square, 2 half 4=2 2
0	$C^{2} = 3.5^{2} + 3.5^{2}$ $C = \sqrt{3.5^{2} + 3.5^{2}}$ $C = \sqrt{24.5}$ $C = 4.9 = \sqrt{5}$	$c = \frac{1}{\sqrt{2}}$ $= \frac{1}{\sqrt{2}}$ $= \frac{1}{\sqrt{2}}$ $= \frac{1}{\sqrt{2}}$

PoW#4 – This was done by one of my focus students – a "struggling learner"!



	First Let's Find out what the Dimensions of the	uores res rhau nght shit shit shit shit shit shit shit sh
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The Short	ant the super can see from my administers the leg of the autside thangle is the same length as the square that is smaller. Using this line that means the next smaller square should be 2". Then if the apply the logic again, I get 1.5" as the side length of the dirth red square. This means that the dimensions of the dark red squares in the middle	PoW#4 - This student made a predictions about a repeated pattern and then tested out his theory using

the losic again, I get 15" as the same length as the square that is smaller. Using this line, that means the next smaller square should be 5". Then if we apply the losic again, I get 1.5" as to the side length of the dark red square. This means that the dimensions of the dark red squares in the middle should be 1.5" by 1.5" if the quilt is 7 feet by 7 feet. Just to make sure	student n predict about repea pattern then testa his theory anoth strategy
Let's Uneck Continue: $0^{2} b^{2} = c^{2}$ $0^{2} a^{2} b^{2} = c^{2}$ $18 = 5e^{2}$ $18 = 5e^{2}$ $19 = c^{2}$ $10^{2} = c^{2}$	very str syster understa of th relation found u triangle was the
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Overall, I am very happy with the results. Students are showing their work, which is important because if they can explain what they are doing it is more likely that they understand – they are demonstrating a chain complex, which may be the purest form of thinking (Vygotsky, 1934/2012). Some of them are collaborating, using "scrap paper", and are then re-writing their work to make their logic easier to understand. Most of the students are able to explain their work orally when asked to explain their reasoning. I noticed this in class and I also noticed that if the student's work had errors, many of the students identified their mistake in the process of explaining their reasoning to me through conversation.

Additionally, I feel that the problems I chose were fairly suitable to match the curriculum we were covering. The first two were not particularly differentiated to provide a variety of ways that they could be solved, however, the last two were very differentiated and the same final answer could be reached using different strategies. For this grade 10 class, I only provided 1 PoW per week, however, with a grade 9 class that I also am currently teaching, I tried to differentiate in another way.

The grade 9 class is a very mixed group regarding background knowledge, learning disabilities, attendance issues, and general work habits. After their first two PoW's, I realized that many students were not completing their assigned PoW's. A co-worker stated that she felt the problems were too hard for this group and that the PoW's were discouraging the students from doing any work at all. In order to reach both the high and low attaining students, a teacher can differentiate by task or by outcome (Boaler, 2002). I had already differentiated by outcome, but after reflecting on the conversation with my coworker, I reluctantly decided to try and differentiate by task as well. I would offer two choices to these students. Each PoW would need to apply similar mathematical reasoning, however, one question was easier and could be solved using either strategies learned in class from past grades (example: algebraic equations, diagrams and illustrations, charts, or guess and check could all be used to solve the easier question). It was extra work for me to find similar but different questions that were easier but still challenging. I offered students the opportunity to complete either one, or both, and was impressed with the increase in submissions. The grade 9 submission rate is still not as high as my grade 10 class, but since offering two PoW choices, it has increased. Even though it is extra work, I will continue to offer this particular class two options for their assigned PoW's.

Chapter 21: I get it

I made a mistake. At the start of this research project I listed (word) problem solving as one of the things we learn to do. But it isn't. I think it's **everything** we do in this class. The entire course is really all about problem solving. I've also come to recognize that I see more value in "word" problems – ones that have a story attached to them in some way and that aren't easily solvable. Additionally, I find problems that can be solved in different ways the most intriguing.

The engines of mathematical knowledge are not axioms, but problems (Cellucci, 2017)

I realize now that this is why over the past ten years I have moved further and further away from the classic lessons of notes followed by seatwork. I do see value in this approach, but generally speaking, I find myself getting antsy with it. Whereas with problems, there's always something interesting or slightly different to solve. Questions can come from many sources including textbooks, and even questions that appear to be close ended can be manipulated to reflect open ended scenarios which then can reach a broader range of students (Small & Lin, 2010). Our district's math numeracy teachers are also an excellent resource for ideas and activities (Garneau, 2016; Hunter, 2014). Regardless of the source, once the student is engaged, the chosen question has reached its goal. Skills, techniques, and formulas are really just supporting actors to the word problem.

I teach the grade 10 Pre-calculus and Foundations course using a spiraled approach. We start with the abstract concept and move towards systemic understanding through a carefully planned set of experiences. As "direct teaching of concepts is impossible and fruitless" (Vygotsky, 1934/2012, p. 159), this means that the tricky part in choosing Problems of the Week, is in choosing questions that reflect conceptually where the learner is at, yet is still challenging enough to encourage a deeper sense of curiosity and understanding. Boaler (2002, p.181) found that if students were engaged, all levels and all classes of students could develop a conceptual understanding. The chosen problems need to engage the student's sense of wonder.

The capacity to wonder is also an attitude toward experience (Sinclair, 2006, p. 130)

Chapter 22: Time slips by

The observation period for gathering data for my action research initially planned to include four but could span up to six weeks. With the school not in session for the province's Family Day holiday plus one Professional Development day, I would have up to 28 classes with my students. Since the first few days are filled with classroom setup and review, the time has now dropped to about 25 classes. With extreme weather, came another 3 days lost to school closure. Added to this were another 3 days lost to an assembly, a course planning visit from the counselors, and one day that I was off due to illness. Suddenly we were down to only 19 classes with spring break at the end of this period. In addition it was suddenly report cards, and I found I had only formally assessed several of the curricular content for this course. While the PoW's are part of the student's mark, since I mark everything else based on skill mastery, I didn't want to put too much emphasis on these questions for this report card. Fortunately, I was able to squeeze in few more official assessments just before marks cut off. Thankfully, I was able to extend my observations another few days post spring break and was rewarded with valuable information gathered both in observations, informal interviews, and a written survey.

From the observations and informal interviews, I learned several things about my focus group. Student A gets most involved when she is working with her close friends. She feels more confident and is more engaged in brainstorming ideas. Student B enjoys working with her friends but is comfortable working with others as well. Student C enjoys working with others if they have a good working relationship. He enjoys working with others and are supportive but who are comfortable challenging his ideas. I have noticed on several occasions that when he did not know how to begin, he appeared frustrated, and would be quiet for several minutes. Sometimes this led to vocal self-talk with comments such as "I'm probably wrong", "I doubt that this is right", or "I just don't understand". Unfortunately, his seating partner for the first half of observations was away and when we switched seats, his new partner missed many classes. As a result, when we did partner work, he either worked alone or joined various groups of 2. Student D has missed many classes and I have made minimal observations on his progress or involvement in word problems.

From the written survey, I learned many things about my students and about myself. Similar to the first week of class, I presented a final written survey to students and asked them to fill it out with as much detail as they wanted. Most surveys took advantage of the space for "comments" and I found their comments and reflections insightful and inspiring.

In the first question, I wanted to determine which questions they had connected with the most. I was pleased to see that it was the PoW's that stood out the most for students.

Math Survey of the Week Which question (either from our in class problems, homework, or the PoW's) has been your favorite so far? Why?

Students responses: My favorite question so far has been from:

An Assessment
1/27 surveys
or
4%

Homework 0/27 surveys or 0% An In Class Problem 9/27 surveys or 33% One of the Problems of the Week 17/27 surveys or 63%

Student Comments:

The PoW's can be challenging which makes me want to work harder.

The PoW's are challenging and fun.

My favorite:

PoW's actually made me think.

In the second and third questions I wanted to see how their perception of problem solving had changed,

or for that matter, if it had changed at all.

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	l about word prob Circle your choice	l about word problems now ? Circle your choice.	l about word problems now ? Circle your choice.

Students results: How I feel about (word) problems before this class compared to now:				
Improved	No change	Decline		
19/27 surveys	7/27 surveys	1/27 surveys		
or	or	or		
70%	26%	4%		

I am very pleased that the general perception indicates an increased interest in problem solving. Upon reflection, it would have been interesting to compare results if the "prior" question was given at the start of the course.

Questions 4 through 6, has helped me to understand my student's needs and perspectives, but also gave me insights into my own teaching.

4) Do you prefer to work on challenging problems:

(Select your top 3 choices, and put them in order of preference)

- 1 = 1 st choice $2 = 2^{\text{nd}} \text{ choice}$ $3 = 3^{\text{rd}} \text{ choice}$
- In class, while collaborating with a partner

In class, working independently

Outside of class, while collaborating with someone (If so, who?

Outside of class, in LST

Outside of class, working independently

Other:

5) Our class completion rate of the PoW's is very high. Why do you suppose that this is?

6) Has your opinion towards mathematics changed since the start of the semester? If so, how?

7) Anything else that you would like to share with me?

Some of the student's explanations as to why they think our completion rates are high are as follows. I find it interesting that many of their comments tie into the emotional engagement as to the pleasurable feeling of accomplishment.

The questions are appealing and they are something to brag about

Probably because people find them challenging and take pride in completing a challenge.

I feel that because we all help each other and our class has a very positive attitude towards learning.

They are fun to do.

We have a lot of time and they give everyone a challenge.

Probably because they might get a zero and they would rather get a perfect mark.

Because they are challenging but are also engaging to do, plus we get a new one every week and the question changes dramatically in terms of difficulty. I like the PoW's because I like the feeling of solving hard problems.

There is no reason to not submit as we can get help from our peers.

Because they are exciting and after you look at the question, everyone wants to try it because it looks so fun.

Because they are interesting and once you start you are determined to finish.

There's a lot of time to complete it so there's no excuse.

It's because the questions are fun and interesting and they make you think about more than one concept.

It's because they are a challenge and that makes them fun to do and it feels good when you get it.

From the survey I had another opportunity to reflect on my own teaching style, values, and areas that I want to continue to work on. For example, I don't do a lot of formal notes, but one student requested that we document the processes we've learned a little more often. I can do that. Many students commented that they use of the giant whiteboards were incredibly useful and a few commented that they learned more from this partner approach than if they had of worked on the question independently. Another student commented

that they really liked the open seating plan that we have switched to – they students chose their table group (now in groups of 4 but still working in partners). I feel that I have set the tone of the class in which we all work together and as long as they make good choices about where they sit, I am fine with it. I did find it interesting that one student initially sat next to his friends, who are all struggling, but has since moved to another group which should be a better influence on helping him to improve.

Chapter 23: What's next?

This action research has been very beneficial to my improving my own teaching. In one regard, I am much more aware of individual students and am also more interested in getting feedback from the students. For example: after the official observation period was over, this same class did puzzles to demonstrate and practice essentially 64 questions on perfect square trinomials. While I haven't used the puzzle with this specific topic before, I have used it for other topics. What happened at the end occurred as a direct result of doing my action research. At the end of the class, I took a few minutes and asked the students for feedback to determine if they felt more confident in factoring these special cases. It appeared that all students felt that they had a better understanding and would retain the information. When asked for votes regarding the 64 questions, that they had to create and solve, versus 64 text questions that could have been given to them, 22 out of 28 students said that they felt that the puzzle activity was more effective and they would "retain the information longer". Six students said it didn't matter, both text practice and the puzzle creation were equally effective. No students reported that they felt they would learn more by independently working on text questions. The next day when the students had to solve the puzzles created by their peers, I also asked for feedback at the end and many students commented that they became much more aware of common errors that could occur. When I assigned them that evenings homework, which I don't do very often, two students actually called out "yes!" because they were excited to go home and practice this style of question. I'm not sure if I've never had any student get excited when I assign homework, especially 50 questions of rote practice.



This group of students seems to work together very well. They are supportive and are encouraging to each other. It may just by chance a great group of students, but I like to think that I have set the tone for a collaborative hard working environment. In these photos, some of the students in the class, doing a team building activity called "the floating stick" prior to working on a new concept

This project has also made me much more aware of potential problems that I could use for my PoW's. In the article "Wondering about wonder in mathematics", a counterintuitive probability situation is exposed (Zazkis & Zazkis, 2014). I am confident that it would be an anomaly to what the students currently think they know and I will tie this into an upcoming PoW. This same article has introduced me to "Simpson's paradox" and while I am sure I've heard about and possibly even learned about in a statistics courses from University, however, I certainly didn't remember it. Now that I am starting to appreciate it, Simpson's paradox is really very interesting. It seems counterintuitive yet, in a puzzling way, it makes sense when you really look at. I will definitely be incorporating this into our problem solving atmosphere when we examine the statistics part of our course.

Zazkis & Zazkis (2014) argue that genuine surprise is the main catalyst for wonder in mathematics. A result is both unexpected and unexplained and this "wonder at" evolves into a "wonder why" (Zazkis & Zazkis, 2014). In the few weeks that we've had together as a class, I think we have just scraped the surface of exploring wonder. I've seen students get excited to make predictions about patterns, test them out, and then use other strategies to confirm their ideas. We haven't been shocked at any of our answers yet and I will need to focus on finding problems that are anomalies.

One other thing that the PoW project has made me think about is the need for me to mediate the problem solving process. I have recently re-discovered the following guide to problem solving and plan to use it in upcoming lessons – I have seen it, or variations of it, previously but had forgotten about this layout:

What problem are you trying to figure out?	What guesses do you have?
What do you already know from the problem?	What do you need to know to solve the problem?
What is your conclusion? How did you reach that	conclusion?

Source: http://robertkaplinsky.com/problem-solving-framework/

Chapter 24: Three last items

Item #1

The last two questions on the final in-class survey had comments that surprised me, but were a good note

to end this project on. They made me feel good about teaching.

Question: Has your opinion towards mathematics changed since the start of the semester? If so, how?

I've like math a bit more because of the PoW's and the whiteboard problems

I am starting to understand math

I look at questions differently now

Since we have done concepts so many times, I feel that I definitely have improved and understand therefore I like math

I've started to enjoy math more

I used to not like math because it was boring. Now I like math because it is fun and there's lots to discover

I have a better understanding of math and I love trig now

Math is harder than last year and I am finding it hard to keep up

I am beginning to understand math and see that it actually is a universal language

Question: Anything else you would like to share with me?

Honestly I feel that you are a very great teacher. You are very helpful with questions in class.

I like how you spend a lot of time on the concepts because it really helps me and everyone understand it fully

Math is really fun the way you teach it

I enjoy this class a lot and I also like your teaching style

Before now I feel like my math teachers were just doing their job. Mrs. Brockway is the first enthusiastic math teacher I've had and it's a nice change.

Item #2

The following exchange happened during the write up of this paper. It was at the end of a class spent practicing factoring perfect square trinomials.

Student: Uh Mrs. Brockway

Me: Yes?

Student: I used to not like math but I'm starting to really enjoy it

Me: That's so nice to hear. I love that you're feeling this way and you've just made my day!

Me (inner voice): Yahoooo!!!

Item #3

On a last and final note, our 2017 Challenge is still in progress and while I have doubts that it can even be done, my students are still thinking about it and that's really what I want my class to be about.

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